SOLUTION

Phy201

Electricity & Magnetism

Final, Fall 2006

16 February2007

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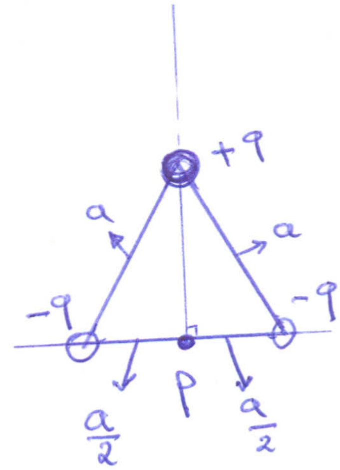
NAME	**ID#**

10 problems, 32 points (2 bonus point)
120 minutes of allocated time

Good Luck to All

$\boxed{3 \text{ PTS}}$

1. Three charges of equal magnitude q reside at the corners of an equilateral triangle of side length a (Fig. P23.63). (a) Find the magnitude and direction of the electric field at point P, midway between the negative charges, in terms of k_e, q, and a. (b) Where must a $-4q$ charge be placed so that any charge located at P will experience no net electric force? In part (b) let the distance between the $+q$ charge and P be 1.00 m.



a) Due to symmetry E_- and E_- cancel each other (due to $-q$'s)

The $+q \Rightarrow E = \frac{kq}{r^2} = \frac{kq}{\frac{3a^2}{4}} = \frac{4kq}{3a^2}$ pointing down

$\vec{E} = \frac{4kq}{3a^2}(-\hat{j})$

b) If $F=0 \Rightarrow E$ at $P = 0$ ∴ $-4q$ must be

above $+q$ on y axis

$E = 0 = -\frac{kq}{1^2} + \frac{k(4q)}{(1+y)^2} \Rightarrow$

$0 = -1 + \frac{4}{(1+y)^2} \Rightarrow y^2+2y-3=0$ → $y=-3$, $y=1$

only $\underline{y=1}$ is acceptable

3 PTS

2. An infinitely long cylindrical insulating shell of inner radius a and outer radius b has a uniform volume charge density ρ (C/m³). A line of charge density λ (C/m) is placed along the axis of the shell. Determine the electric field intensity everywhere.

$r<a$ $\quad \phi = q_{in}/\epsilon_0$ 1

$E\, 2\pi r L = \lambda L/\epsilon_0$

$E = \dfrac{\lambda}{2\pi r \epsilon_0}$ 1

$a<r<b$ $\quad E \cdot 2\pi r L = \left[\lambda L + \rho\pi(r^2 - a^2)L\right]/\epsilon_0$

$E = \dfrac{\lambda + \rho\pi(r^2-a^2)}{2\pi r \epsilon_0}$

$r>b$ $\quad E \cdot 2\pi r L = \left[\lambda L + \rho\pi(b^2-a^2)L\right]/\epsilon_0$ 1

$E = \dfrac{\lambda + \rho\pi(b^2 - a^2)}{2\pi r \epsilon_0}$ 1

3PTS

3. Calculate the electric potential at point P on the axis of the annulus shown in Figure P25.49, which has a uniform charge density σ.

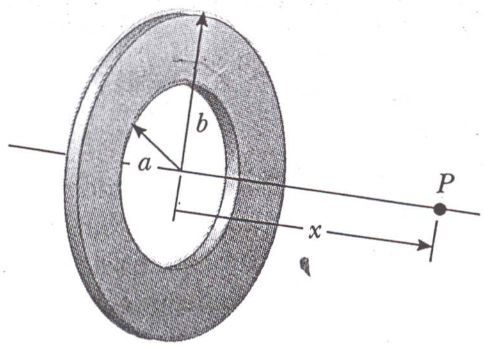


FIGURE P25.49

$$dU = \frac{k\,dq}{\sqrt{r^2+x^2}} \qquad dq = \sigma\, dA = \sigma 2\pi r\, dr$$

$$dU = \frac{2k\pi\sigma\, r\, dr}{\sqrt{r^2+x^2}}$$

$$V = 2k\pi\sigma \int_a^b \frac{r\,dr}{\sqrt{r^2+x^2}}$$

$$V = 2k\pi\sigma\left[\sqrt{x^2+b^2} - \sqrt{x^2+a^2}\right]$$

$\boxed{4PTS}$

4. A parallel-plate capacitor in air has a plate separation of 1.50 cm and a plate area of 25.0 cm^2. The plates are charged to a potential difference of 250 V and disconnected from the source. The capacitor is then immersed in distilled water. Determine (a) the charge on the plates before and after immersion, (b) the capacitance and voltage after immersion, and (c) the change in energy of the capacitor. Neglect the conductance of the water.

ORIGINALLY $C = \epsilon_0 A/d = \frac{Q}{V} \Rightarrow$

a) charge is same before immersion

$$Q = \frac{\epsilon_0 A V}{d}$$

$$Q = \frac{8.85\times10^{-12} \times 25\times10^{-4} \times 250}{1.5\times10^{-2}} = \boxed{3.7\times10^{-10}\ C}$$

b) FINALLY:

$$C_f = \frac{k\epsilon_0 A}{d} = \frac{Q}{V_f}$$

$$C_f = \frac{80 \times 8.85\times10^{-12} \times 25\times10^{-4}}{(1.5\times10^{-2})} = 1.18\times10^{-10}\ C$$

$$V_f = \frac{Qd}{k\epsilon_0 A} = \frac{\epsilon_0 A V d}{k\epsilon_0 A d} = \frac{V}{k} = \frac{250}{80} = \boxed{3.12\ V}$$

c) $U = \frac{1}{2} C V^2 = \frac{\epsilon_0 A V^2}{2d}$

$$U_f = \frac{1}{2} C_f V_f^2 = \frac{\epsilon_0 A V^2}{2dk}$$

$$\Delta U = U_f - U_i = -\frac{\epsilon_0 A V^2 (k-1)}{2dk} = \boxed{-4.52\times10^{-8}\ J}$$

$\boxed{3\ \text{PTS}}$

5. A circular disk of radius R and thickness d is made of material with resistivity ρ. Show that the resistance between points a and b (Fig. P27.18) is independent of the radius and is given by $R = \pi\rho/2d$.

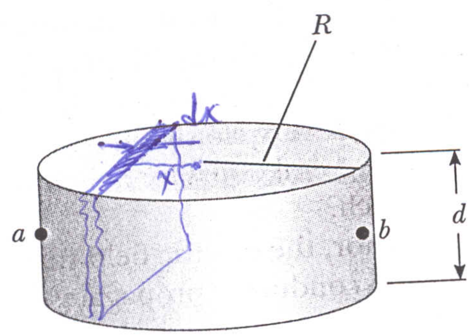


FIGURE P27.18

$$2\sqrt{R^2-x^2} \Rightarrow dR = \frac{\rho L}{A} = \frac{\rho\,dx}{2d\sqrt{R^2-x^2}}$$

$$R = \int dR = \int_{-R}^{+R} \frac{\rho\,dx}{2d\sqrt{R^2-x^2}}$$

$$= \frac{\rho}{2d}\sin^{-1}\frac{x}{R}\Big|_{-R}^{+R} = \frac{\rho}{2d}\left(\frac{\pi}{2}-\left(-\frac{\pi}{2}\right)\right] = \frac{\rho\pi}{2d}$$

$\boxed{3\text{ PTS}}$

6. (a) Find the potential difference between points a and b in Figure P28.25. (b) Find the currents I_1, I_2, and I_3 in Figure P28.25.

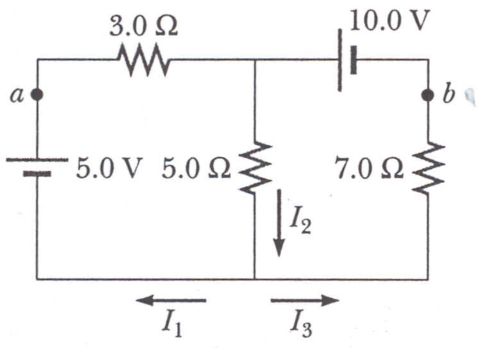


FIGURE P28.25

$I_2 = I_1 + I_3$ 1

$5 - 3I_1 - 5I_2 = 0$ 1

$10 - 5I_2 - 7I_3 = 0$ 1 $\Rightarrow$

$I_1 = 0.141$ A

$I_2 = 0.915$ A 1

$I_3 = 0.774$ A

$V_{ab} = V_b - V_a = 0 - [10 + 3 \times 0.14] = \boxed{-10.42\text{ V}}$ 1

$\boxed{3\text{ PTS}}$

7. A power supply has an open-circuit voltage of 40.0 V and an internal resistance of 2.0 Ω. It is used to charge two storage batteries connected in series, each having an emf of 6.0 V and internal resistance of 0.30 Ω. If the charging current is to be 4.0 A, (a) what additional resistance should be added in series? (b) Find the power lost in the supply, the batteries, and the added series resistance. (c) How much power is converted to chemical energy in the batteries?

a) $\mathcal{E} - I\sum R - (\mathcal{E}_1 + \mathcal{E}_2) = 0$

$40V - 4 * [2 + 0.3 + 0.3 + R] - (6 + 6) = 0$

$R = 4.4\Omega$

b) $P = I^2R = 4^2 * [2 + 0.3 + 0.3 + 4.4] = 112\,W$

c) $P = I(\mathcal{E}_1 + \mathcal{E}_2) = 4 * (6 + 6) = 48\,W$

3PTS

8. A voltage $v = (100\text{ V})\sin \omega t$ (in SI units) is applied across a series combination of a 2.00-H inductor, a 10.0-μF capacitor, and a 10.0-Ω resistor. (a) Determine the angular frequency ω_0 at which the power dissipated in the resistor is a maximum. (b) Calculate the power dissipated at that frequency. (c) Determine the two angular frequencies ω_1 and ω_2 at which the power dissipated is one-half the maximum value.

$L = 2H \quad C = 10^{-4} F \quad R = 10\Omega \quad V = 100 \sin \omega t$

a) $\omega_0 = \frac{1}{\sqrt{LC}} = 224$ Rad/s

b) $P = \frac{V^2}{2R} = \frac{(100)^2}{2 \times 10} = 500\,W$

$\frac{1}{2}$

c) $I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (L\omega - \frac{1}{C\omega})^2}}$ and $I_{max} = \frac{V}{R}$

$I^2 R = \frac{1}{2} I_m^2 R$ or $\frac{V^2}{Z^2} R = \frac{1}{2} \frac{V^2}{R^2} R$

$\frac{1}{2}$

$Z^2 = 2R^2$

$R^2 + (\omega L - \frac{1}{C\omega})^2 = 2R^2$

$L^2C^2\omega^4 - (2LC + R^2C^2)\omega^2 + 1 = 0 \Rightarrow$

$\frac{1}{2}$

$\omega^2 = 51130$ and 48894

$$\boxed{\begin{aligned} \omega_1 &= 221 \text{ Rad/s} \\ \omega_2 &= 226 \text{ Rad/s} \end{aligned}}$$

$\frac{1}{2}$

3 PTS

9. An ac voltage with an amplitude of 100 V is applied to a series combination of a 200-μF capacitor, a 100-mH inductor, and a 20.0-Ω resistor. Calculate the power dissipated and the power factor for a frequency of (a) 60.0 Hz and (b) 50.0 Hz.

a) $f = 60\,Hz$ $\omega = 377$ rad/s $\omega L = 37.7\,\Omega$ $\frac{1}{\omega C} = 13.26\,\Omega$

$Z = \sqrt{R^2 (L\omega - \frac{1}{C\omega})^2} = 31.58\,\Omega$

$I_{MAX} = \frac{V_{MAX}}{Z} = \frac{100}{31.58} = 3.17\,A$

$P_{AV} = I^2_{RMS} R = \frac{I^2_{MAX}}{2} R = 100.3\,W$

$P_{AV} = \frac{1}{2} I_m V_m \cos\phi \Rightarrow \boxed{\cos\phi = 0.633}$

b) $f = 50\,Hz$ $\omega = 314$ $\omega L = 31.4$ $\frac{1}{\omega C} = 15.9$

$Z = 25.3\,\Omega$

$I_{MAX} = \frac{V_{MAX}}{Z} = \frac{100}{25.3} = 3.95\,A$

$P_{AV} = I^2_{RMS} R = \frac{I^2_{MAX}}{2} R = 156\,W$

$P_{AV} = \frac{1}{2} I_m V_m \cos\phi \Rightarrow \boxed{\cos\phi = 0.79}$

4PTS

10. A long piece of wire of mass 0.10 kg and length 4.0 m is used to make a square coil 0.10 m on a side. The coil is hinged along a horizontal side, carries a 3.4-A current, and is placed in a vertical magnetic field of magnitude 0.010 T. (a) Determine the angle that the plane of the coil makes with the vertical when the coil is in equilibrium. (b) Find the torque acting on the coil due to the magnetic force at equilibrium.

a) $\mu = NAI$

$\mu = \frac{4}{4d} d^2 I$ at θ

$\Sigma \tau = 0 = (\vec{\mu} \times \vec{B}) - (\vec{r} \times m\vec{g})$

$0 = \frac{Ld}{4} IB \sin(90-\theta) - \frac{d}{2} mg \sin\theta$

$\frac{mgd}{2} \sin\theta = \frac{ILBd}{4} \cos\theta$

$\theta = Tg^{-1}\left(\frac{ILB}{2mg}\right) = 3.97^\circ$

b) $\tau_m = \frac{ILBd}{4} \cos\theta = \boxed{3.39\ mN \cdot m}$